# Analysis of Convertible Bond Price Sensitivity Subject to Interest Rate and Other Variables

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# Chapter 1

# Introduction

## 1.1 An Introduction to Convertible Bonds

When creating an investment portfolio, portfolio managers choose from three main classes; equity, bonds and hybrid financial instruments. Convertible bonds would be classified as hybrid financial instruments and are sometimes overlooked by investors. Essentially a convertible bond is a bond that can be converted into shares at a particular date. As a hybrid financial instrument, convertible bonds are more difficult to value than the other investment classes. Convertible bonds can be distinguished by the following characteristics; they have a fixed maturity, a fixed repayment, fixed interest payments and a conversion privilege.

Companies who wish to raise capital can do so by issuing equity, bonds or convertible bonds. When a company issues equity, no repayment is needed for the raise in capital. Equity holders of a particular company have the right to share in decisions made within the company. When a company chooses to issue bonds in order to raise capital then the full amount needs to be repaid on a fixed date. Now if a company were to issue convertible bonds then the company will take on debt capital that will be fully or partially converted into share capital during the life of the convertible bond. By issuing convertible bonds the company will effectively receive a share capital increase that is spread over time. Capital that is raised through convertible bonds does not need to be repaid and thus this is one of the main reasons why companies would choose to do their financing through the use of convertible bonds.

There are many types of investors who can profit from convertible bonds. The main advantage of convertible bonds, from an investor's stand point, is that they offer the high upside potential of equity while strongly limiting downside risk. Both issuers and investor of convertible bonds you should be aware of which variables convertible bonds are sensitive to and have some understanding of how the convertible bond price reacts when these variables are adjusted. By constructing a sensitivity report one could easily be able to understand precisely how the value of a convertible bond changes when certain variables are altered.

## 1.2 Properties of Convertible Bonds

Convertible bonds are moderately simple in concept; A convertible bond can be seen as a financial instrument that is made up of two distinct parts; the corporate bond part and the option part. The corporate bond part functions exactly the same as standard corporate bond would, while the option part adds the additional property that allows the conversion of the instrument into equity.

Since convertible bonds share the characteristics of both bonds and stocks within the same single financial instrument we can see that the price of convertible bonds are affected by both interest rates and share prices. We note that when the share price is relatively low compared to the conversion price, then it is quite unlikely that the convertible bond will be converted thus it is effectively just a straight bond where share price has no affect. When the share price is relatively high in comparison to the conversion price, then there is a high certainty that the convertible bonds will be converted into shares. In the latter case, the convertible bond price will be the share price times the conversion ratio. The conversion feature of a convertible bond closely resembles a call option on the underlying stock. This feature becomes more valuable as the volatility of the stock increases, thus increasing the convertible bond price.

There are many types of investors who can profit from convertible bonds. The main advantage of convertible bonds, from an investor's stand point, is that they offer the high upside potential of equity while strongly limiting downside risk. The banks that underwrite convertible bonds make their profit from the transaction fees. As for long term investors, convertible bond portfolios are known to outperform the more traditional bond and equity portfolios. Equity investors, who want to capture the upside potential of equity prices, can acquire convertible bonds that are deep in the money so that they can capitalize on the gains in equity while also receiving coupons as an additional income. The coupons paid by a convertible bond are in general higher than the dividends that the underlying stock would have paid.

As a fixed income investor you would typically buy convertible bonds deep out of the money so that you can receive the yield similar to the corporate bond counterpart while also being able to capitalize on the upside potential of the underlying share price and fixed income payments from the coupons. If you are a hedge fund manager, you might want to capitalize on convertible bond arbitrage.

Convertible bond arbitrage is mostly adopted by hedge fund managers; it involves going long on the convertible bond and shorting the underlying stock according to the delta hedging strategy. If the stock price of the underlying stock declines then the convertible bond is similar to a traditional bond thus limiting your downside movement, since the strategy involves you shorting the stock you can capitalize on the stock decline. If the stock price of the underlying stock were to rise then the convertible bond adopts similar properties to equity and thus appreciates in value as the stock price rises.

There is no active market for convertible bonds; instead most convertible bonds are traded by appointment and not on the exchange floor. Brokerage firms normally bring buyers and sellers together so that they can meet and come to a trade agreement, depending on the size order, market conditions and other factors this process can take a few minutes to several days. The issuance of convertible bonds by a company can be considered a conditional issuance of equity. If a company?s growth opportunity increases then so would the company?s stock price which would lead to the conversion of the convertible bonds, providing the company with an inflow of equity when it needs it most.

The concept of convertible bonds might be relatively simple to understand, but the evaluation of a convertible bond is fairly complicated. There is, however, an intuitive pricing formula which is merely the linear function of a corporate bond summed with the value of a call option representing the conversion privilege. This pricing formula may not be accurate by any means, but serves as a good illustrative pricing structure for our sensitivity report.

# Chapter 2

# Convertible Bond Pricing Models

## 2.1 The Simple Component model

Our first step is to decide which method to use when pricing a convertible bond. A convertible bond can be seen as a derivative product that is created through the combination of a standard corporate bond and an option. So to price this instrument we will price the corporate bond part and option part separately and then sum the two values in order to get the convertible bond price.

The general pricing formula is as follows; CB=IV+Call So the price of a convertible bond is the summation of IV, the price of the standard corporate bond portion and Call the price of the option portion of the convertible bond price.

Evaluating the price of the standard corporate bond portion will be done by the standard bond pricing formula. The standard formula for pricing a bond is as follows;

$IV = \sum_{(t=1)}^{2T} (C/(1+i+cr)^t) + Par(1+i+cr)^{-T}$

Here we have that the nominal semi-annual coupon amount is represented by C and the par value of the bond by Par. The interest rate is represented by the symbol i and credit risk by the symbol cr.

To evaluate the price of the option portion of the convertible bond price we will use the Black Scholes pricing frame work. The Black Scholes pricing framework makes many assumptions and is not the most accurate way to price an option, but it is effective enough for the illistrative purpose of the sensitivity report. The Black Scholes pricing framework makes the following assumptions;

Constant volatility
Stock price changes smoothly
Constant stock price drift
Constant stock price volatility
Constant short term interest rate
No trading cost
No taxes
Option is exercised at maturity

The formula for the call portion of the convertible bond pricing formula is as follows;

$$Call = S_t R e^{-q(T-t)} N(d_1) - IV e^{-r(T-t)} N(d_2)$$

where;

$$d_1 = \frac{ln(\frac{S_t R}{IV}) + (r - q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Here we have that St is the current stock price, R is the conversion ratio, q is the dividend rate, T is the expiration time, IV is the bond portion price, r is the risk free rate and N() is the normal cumulative distribution function.

Some convertible bonds have certain clauses stipulated in their contracts that might complicate the pricing formula. Some convertible bonds have a soft call clause which allows the issuer to force conversion if the share price, or the average share price, is above a certain barrier over a particular period of time. Incorporating this particular clause into the pricing formula causes many complications to arise.

One such complication arises from using American binary options to account for the call portion of the convertible bond price becoming null on activation of the soft call clause. Some convertible bonds might even have put features that gives the buyer the option to execute a put on the convertible bond back to the issuer.

This simple model does not incorporate these clauses into the pricing formula of the convertible bond. The soft call clause, in particular, dampens the affect of the stock price on the convertible bond price.

There are many alternative and more accurate pricing models that have been developed and should be used when attempting to accurately price a convertible bond. For the comparative purposes of our sensitivity report we will use the basic pricing formula for a convertible bond as a baseline model to compare more accurate models with.

With respect to the practical example under consideration, namely the Shoprite convertible bond contract, there are many clauses and details in the contract that cannot be accurately described by the simple component pricing model. Such details, among others, include dividend threshold clauses, soft call clause and change of control clause. Some of these clauses will have little to no affect of our sensitivity analysis, while other clauses might distort or dampen the effect of certain variables on our model. The details of the Shoprite contract are used in both pricing models that are analysed in this report.

## 2.2 The Tsiveriotis and Fernandes model

Of the many models used to price convertible bonds, the Tsiveriotis and Fernandes model is the most popular among practitioners. This model is able to account for the default risk associated with the unknown future cash payments of a convertible bond. The model views a convertible bond as a contingent claim on the underlying equity, thus the value of the convertible bond is governed by the following Black Scholes equation;

$$\frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u}{\partial S^2} + r_g S \frac{\partial u}{\partial S} - (r + r_c)u + f(u, S, t) = 0$$

where S is the price of the underlying stock, r is the risk free rate, $r_g$ is the growth rate of the stock, $r_c$ is the credit spread that reflects the risk of payoff default, and $f(V, S, t)$ reflects the predetermined external cash flows (cash or equity).

The Tsiveriotis and Fernades model defines for any convertible bond, a related security known as the "cash only part of the convertible bond". This leads to a new formulation of the convertible bond pricing problem. The problem can now be represented by the system of two coupled Black-Scholes equations;

CB: $\frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u}{\partial S^2} + r_g S \frac{\partial u}{\partial S} - r(u - v) - (r + r_c)v + f(t) = 0$

COCB: $\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 v}{\partial S^2} + r_g S \frac{\partial v}{\partial S} - (r + r_c)v + f(t) = 0$

Here $u$ is the value of the convertible bond and $v$ is the value of the cash only convertible bond.

We assume that all external payments to holder are paid in cash, so $f(t)$ will represent cash flows only.

This system of coupled Black Scholes equations must be solved simultaneously with the following conditions;

For conversion at time $t\epsilon[0, T]$ , where T is the time of expiration, we have the following boundary conditions;

$$\begin{cases} u(S,t) \geq RS \\ v(S,t) = 0 \qquad for\; \bar{u} \leq RS \end{cases}$$

where $R$ is the conversion ratio and $\bar{u}$ is the value of the bond before being exercised

There is no explicit solution to the system of coupled Black Scholes equations. So the first step to calculating a numerical solution is the discretization of the partial differential equations in time t and stock price S. There are many well known numerical methods, such as the finite elements method, that can be used to approximate a numerical solution. We will be using the finite differences method because of its simplicity and efficiency.

Before we proceed, we must apply two different coordinate transformations; Let $\tau = T - t$ and $x = ln(S)$. From this transformation, our system of coupled Black Scholes equations is simplified to a system of simple diffusion equations;

CB*: $\frac{\partial u}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2} + (r_g - \frac{1}{2}\sigma^2)\frac{\partial u}{\partial S} - r(u - v) - (r + r_c)v + f(t)$

COCB*: $\frac{\partial v}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial S^2} + (r_g - \frac{1}{2}\sigma^2)\frac{\partial v}{\partial S} - (r + r_c)v + f(t)$

Then the solution $u(x, \tau)$ and $v(x, \tau)$ is discretized over a grid of set points $x_i$ where $i = 1 \dots N$ and $x_i's$ are equally spaced by a fixed distance $h$. We also need to take finite time steps, $\Delta\tau$.

Thus we have $u^k = u(k\Delta\tau)$ and $v^k = v(k\Delta\tau)$.

Using explicit time steps, our partial differential equations are as follows;

$$CB^{\#} = \frac{u_i^{k+1} - u_i^k}{\Delta\tau} = \frac{1}{2}\sigma^2 \frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h^2} + (r_g - \frac{1}{2}\sigma^2)\frac{u_{i+1}^k - u_{i-1}^k}{2h} - r(u_i^k - v_i^k) - (r + r_c)v_i^k + f(k\Delta\tau)$$

$$COCB^{\#} = \frac{v_i^{k+1} - v_i^k}{\Delta\tau} = \frac{1}{2}\sigma^2 \frac{v_{i+1}^k - 2v_i^k + v_{i-1}^k}{h^2} + (r_g - \frac{1}{2}\sigma^2)\frac{v_{i+1}^k - v_{i-1}^k}{2h} - (r + r_c)v_i^k + f(k\Delta\tau)$$

Now at time $\tau = (k + 1)\Delta\tau$, we start with $(u^k, v^k)$. Using equation $CB^{\#}$we are able to calculate $u^{k+1}$.

After checking conditions for $u^{k+1}$, we can use equation $COCB^{\#}$to calculate $v^{k+1}$ and then check the conditions for $v^{k+1}$.

The coupons, which are discrete cash flows, are added to the solution at the coupon dates.

## 2.3 The Shoprite Contract

During March 2012 Shoprite Investments (Proprietary) Limited released a launch term sheet detailing the elements of their convertibles bonds due for release on the 2nd of April 2012. Convertible bonds combine the characteristics of bonds and stocks into one financial instrument. Convertible bonds give the holder the right to exchange the par amount of the bond for common shares at a predetermined conversion ratio during a particular period or to hold the bond until maturity.

The aggregate principal amount of convertible bonds issued by Shoprite was R4700 000 000.00 where the convertible bonds can be purchased in denominations of R10 000 each. The coupon rate is 6.5% per annum convertible semi-annually, as coupon payments are made semi-annually. If the convertible bond is not previously converted, redeemed or cancelled then the issuer will redeem the convertible bonds at their principal amount together with accrued interest. If at maturity the current market price of the underlying Shoprite shares is below the conversion price then the issuer may, at its own discretion, grant to the bondholders the option for settlement to be in cash or ordinary shares. The base conversion price for the convertible bonds is R168.94 per ordinary share.

In the case of extraordinarily high dividends in any financial year, the conversion price will be adjusted accordingly. For the purpose of our analysis we will assume a constant dividend rate of 0.0187 as quoted as by bloomberg.com as the current year's dividend rate for Shoprite stock thus we also make the assumption of no extraordinarily high dividends in any financial year. Shoprite has also incorporated a soft call clause in their contract that allows the issuer to force conversion if the current stock price exceeds a certian level for a particular period of time. Iplimenting this into our pricing formula would add little value to our sensitivity analysis and would dampen the affect of stock price on our model, thus we will not consider this cluase in our model. The bond portion of our pricing model is highly dependent on interest rate and so should be chosen carefully.

Shoprite's issued their convertible bonds at par value and thus we know that the interest rate that they are assuming must be close to that of the coupon rate. A common practiced method of obtaining the effective annual interest rate for a five year period is to use the yield per annum of a five year zero coupon government bond. On comparison we find that an estimated interest rate of 0.065 effective per annum would be sufficiently accurate for our model. In order to choose the volatility rate for our model we notice that the most significant source of volatility from our model is from the stock price variable. Through calculation of the stock price's historical volatility and then with research into Shoprite's current business prospects we find that a volatility rate of 0.0.1609 is sufficiently accurate for our model.

Our pricing model uses the Black Scholes construct when evaluating the movement in stock price. The Black Scholes framework assumes that the stock price follows geometric Brownian motion with constant drift and volatility. Any calculation on stock drift at this point in time would be pure speculation although it is relatively safe to assume a positive drift. For simplicity we will assume zero drift in our stock price. This assumption will better help us to analyse how the volatility affects certain areas without concern of the affect that drift has on the stock price, as drift may distort the affect of the volatility of the stock price at certain barriers.

The credit spread of the convertible bond can be observed directly from the markets. The credit spread is modelled from the corporate bond price of the issuing company and the price of a riskless government bond. From this we find a credit spread of 0.0054 is adequate for our pricing models.

Now that we have collected the values of the parameters needed for the models under observation, we can now proceed to model the value the Shoprite convertible bond under consideration.

# Chapter 3

# Analysis of Component Model

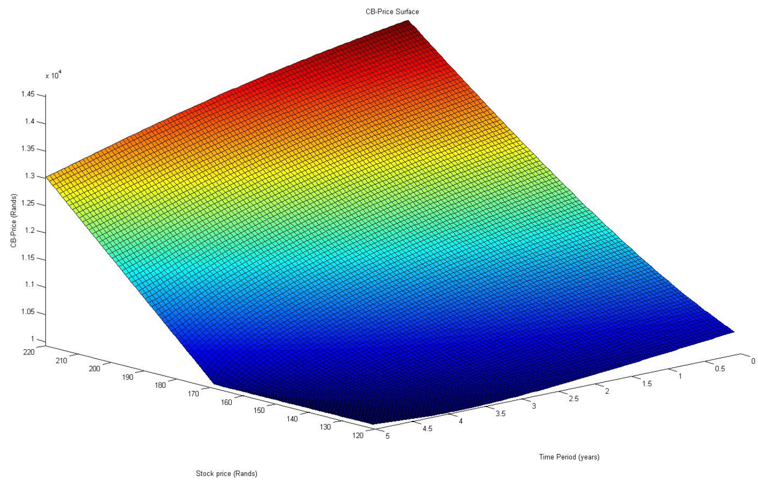
## 3.1 Analysis Introduction

When pricing a convertible bond we should consider the sensitivity of the convertible bond price relative to all of the market variables. We will construct a sensitivity report and perform an analysis so that we may better understand the constraints surrounding convertible bonds.

Convertible bonds have the characteristics of both bonds and stocks therefore the factors that affect the price of these securities are those that are associated with both the bond portion and call option portion of the convertible bond pricing formula. By recording the results of what might happen when these variables are adjusted, we will be able to construct a sensitivity report.

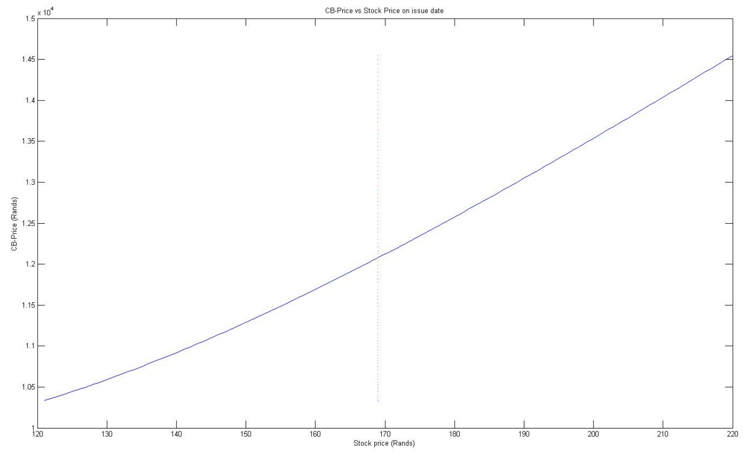
## 3.2 Price Surface

The first analysis we can perform is to construct a price surface;

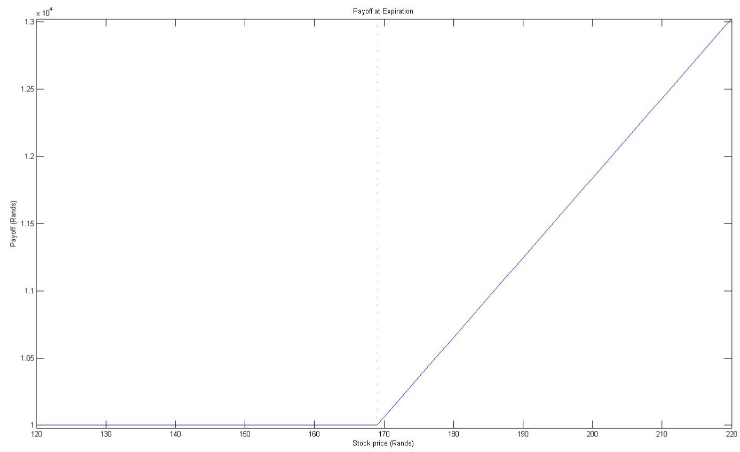


The price surface represents the value of the convertible bond at a given point in time given the stock price at that point in time. We find that at any point in time, the value of the convertible bond is an increasing function of stock price and that the underlying stock price has a lesser effect on the value of the convertible bond further away from the expiration date, opposed to near to expiration.

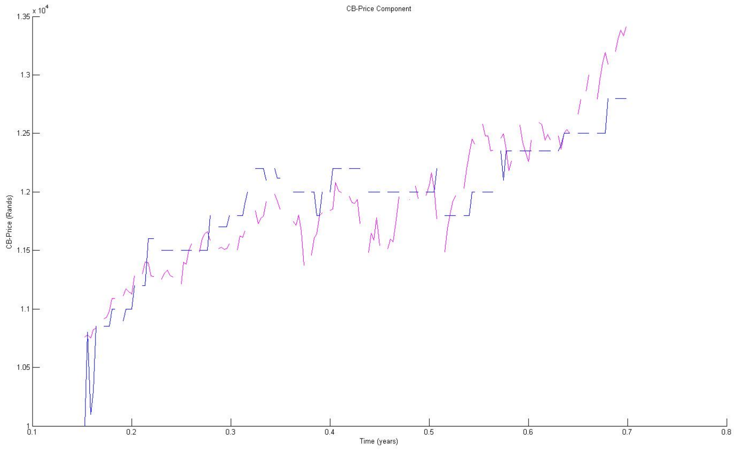
This property becomes more evident when making a comparison of the value convertible bond at issue and the value convertible bond at expiration, relative to stock price. This observation is better depicted in the graphs below, where the conversion price of R168.94 is represented by the vertical dotted line;




With reference to the graph below, we notice that the payoff structure of the convertible bond represents that of a call option plus the par value of the convertible bond;



By collecting the convertible bond trading prices from the 28th of May until the 16th of December, and the corresponding closing prices of the underlying stock, we are able to fit our model to the empirical data. In the plot below, the darker line represents the empirical data;




The regression statistics for this model are as follows;

Quadratic Regression

r-squared=0.8357

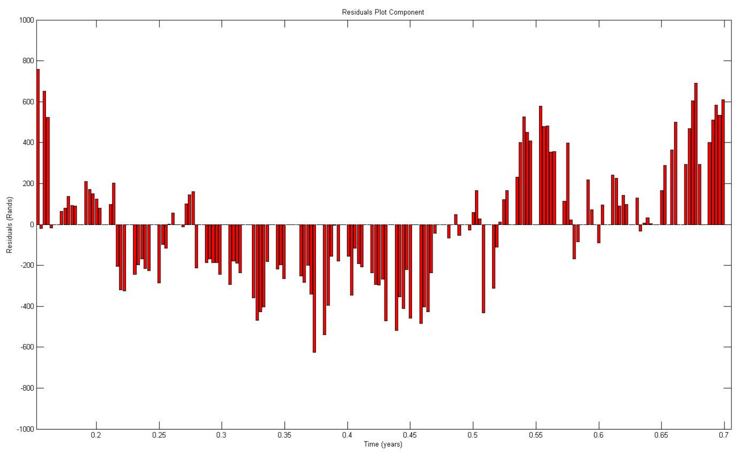
adjusted r-squared=0.8334

Linear Regression

r-squared=0.7472

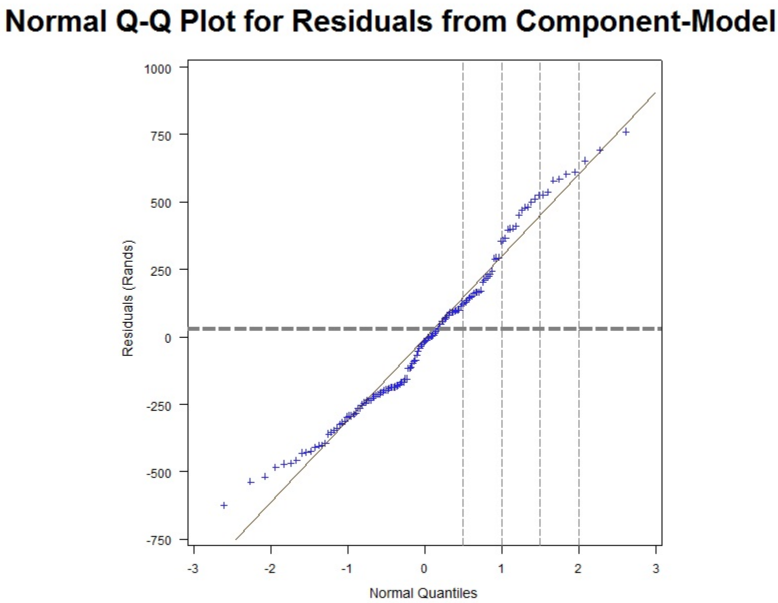
adjusted r-squared=0.7454

The residuals of a model are the estimates of the experimental error obtained by subtracting the observed responses from the predicted responses. The residuals of the fitted model for this time period are as follows;



A well behaved model will have residuals that are roughly normally and approximately independently distributed with a mean of zero and some constant variance.

The qq plot of the residuals show strong signs of normality with a mean that is significantly close to zero;




To confirm our assumption of normality we perform a series of tests for normality. The output is as follows;

**Tests for Location: Mu0=0**

| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | -0.16975 | Pr > \|t\| | 0.8655 |
| Sign | M | -3.5 | Pr >= \|M\| | 0.6110 |
| Signed Rank | S | -349 | Pr >= \|S\| | 0.4651 |

**Tests for Normality**

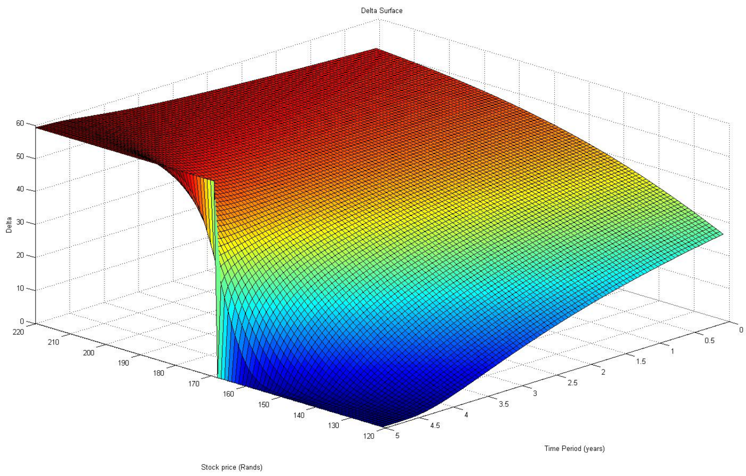
| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.973105 | Pr < W | 0.0076 |
| Kolmogorov-Smirnov | D | 0.100984 | Pr > D | <0.0100 |
| Cramer-von Mises | W-Sq | 0.177255 | Pr > W-Sq | 0.0101 |
| Anderson-Darling | A-Sq | 1.158034 | Pr > A-Sq | <0.0050 |

From this we conclude that even at a 1% significance level we have that the residuals are not from a normally distributed population.

We can now conclude that the residuals contain structure that is not accounted for by the model, thus the model can be further improved.

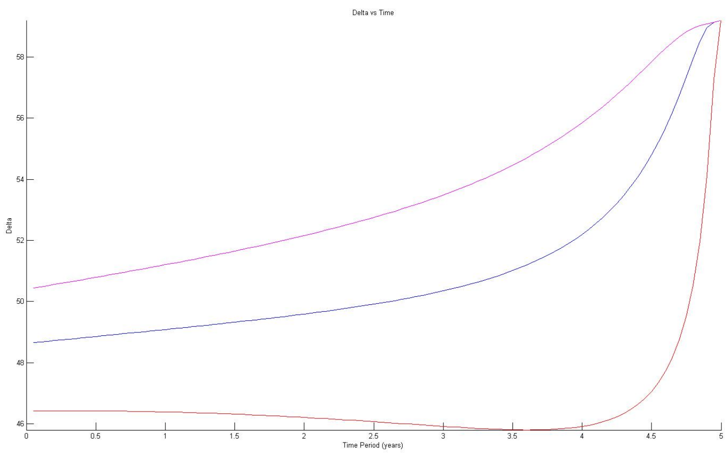
## 3.3 Delta Surface

The call portion of the convertible bond pricing formula has the underlying stock price as a variable. The delta of a security is the rate of change of the price of the security with respect to the underlying asset price. By constructing the delta surface we notice that the delta of the convertible bond is almost entirely dependent on the option portion of the pricing formula. We would only need to delta hedge against the option portion of the convertible bond price.




The delta surface has the shape of a typical call option delta surface.

The three curves below represent the value of delta with respect to time where the value of the underlying equity is either in the money, at the money or out of the money;




(In the Money = magenta, At the Money = blue, Out the Money = red)

## 3.4 Theta Surface

The theta of a portfolio is the rate of change of the value of the portfolio with respect to time with all other variables remaining constant. Our convertible bond pricing formula is a linear combination of bond price and option price therefore, to calculate theta we just need to differentiate the bond portion of the convertible bond pricing formula and add it to the theta equation of the corresponding call option. This is done as follows;

First we adjust the bond pricing formula slightly using a common geometric relation.

$IV = (C)((1-(1+i+cr)^{-2(T-t)})/(i+cr)) + Par(1+i+cr)^{-2(T-t)}$

C= Nominal semi-annual coupon value

i= Effective semi-annual interest rate

Par= The principal value of the bond

T= The expiration time node

t= current time node

cr= Credit risk

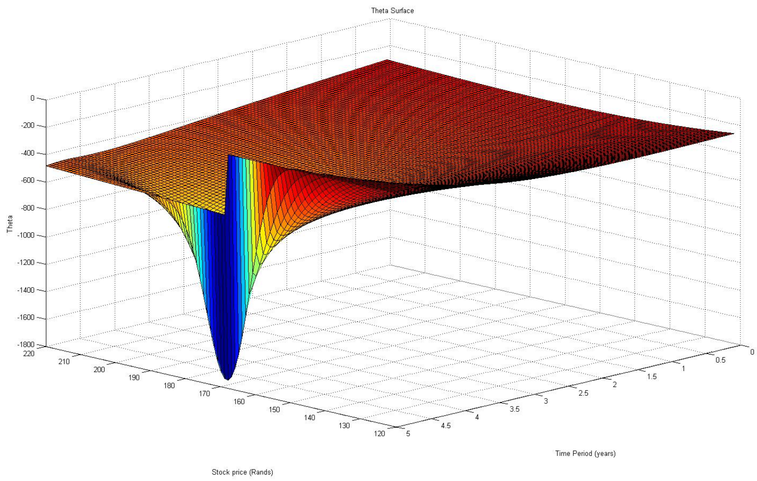
From this point we differentiate with respect to (T-t) and arrive at the following expression;

$\frac{d(IV)}{d(T-t)} = (\frac{C}{(i+cr)})(2ln(1+i+cr))((1+i+cr)^{-2(T-t)}) - (P)(2ln(1+i+cr))((1+i+cr)^{-2(T-t)})$

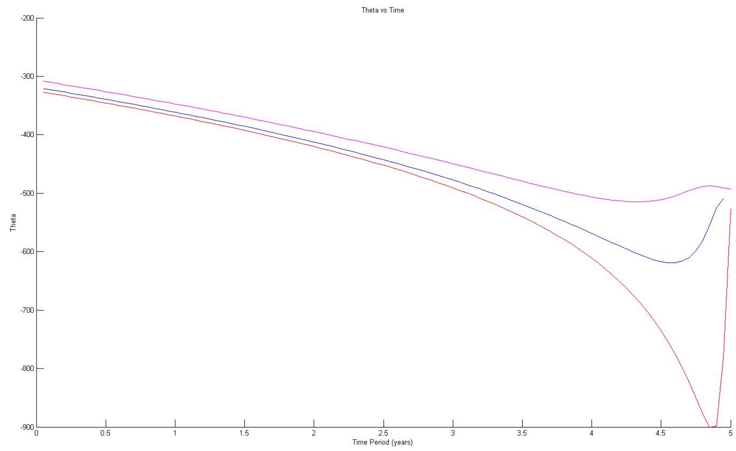
our final expression is as follows;

$Theta = \frac{d(IV)}{d(T-t)} + theta(call)$

Using this expression for the theta of our convertible bond we are able to construct the following theta surface;



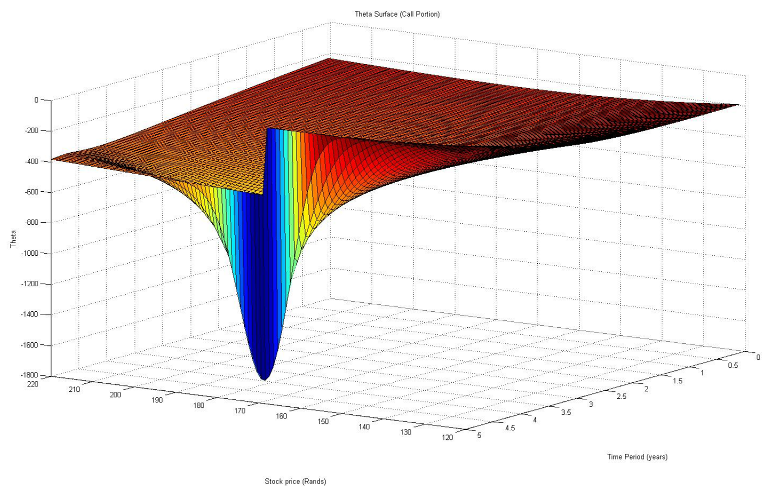
The three curves below represent the value of theta with respect to time where the value of the underlying equity is either in the money, at the money or out of the money;



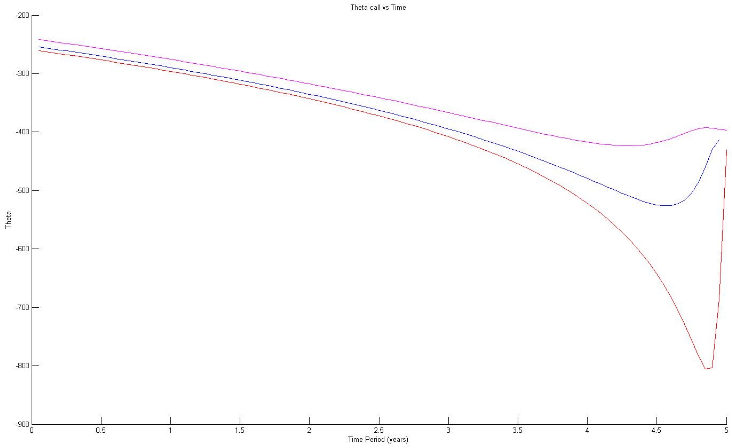

(In the Money = magenta, At the Money = blue, Out the Money = red)

Now by making a comparison of the theta graphs of the option portion of the convertible bond pricing formula to that of the overall convertible bond theta we will be able to analyse the affect that the bond and option portions have on the overall theta.

The theta surface of the call portion of the convertible bond pricing formula is as follows;



There are three typical patterns for the variation of theta with respect to time for the call portion of our convertible bond.




(In the Money = magenta, At the Money = blue, Out the Money = red)

Now by making a comparison of the convertible bond theta surface and the call option portion theta surface we notice that the inclusion of the bond portion theta causes a linear ascension of the convertible bond theta plane of about 7-11 units.

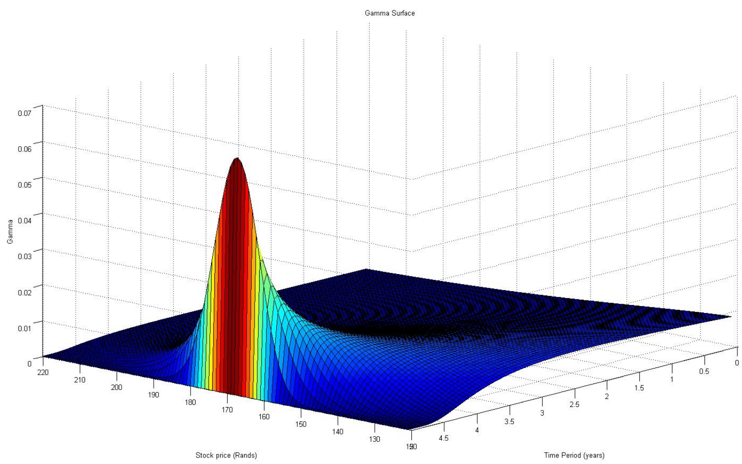
Now by comparing the theta curve graphs depicted above we are able to clearly see the linear affect that the bond portion theta has on the overall theta.

## 3.5 Gamma Surface

The gamma of a portfolio is the rate of change of the value of the portfolio with respect to a change delta with all other variables remaining constant. So a small gamma means that delta changes slowly and therefore it is fairly easy to keep the portfolio delta neutral. However if we were to have a large gamma this would imply that delta has a high sensitivity to price change and therefore it would be fairly difficult to remain delta neutral. Delta hedging assumes that the portfolio price changes in a linear fashion with respect to stock price but this is rarely the case as the relation is normally nonlinear. By hedging against gamma in order to stay gamma neutral we can effectively hedge against the hedging error introduced by nonlinearity.

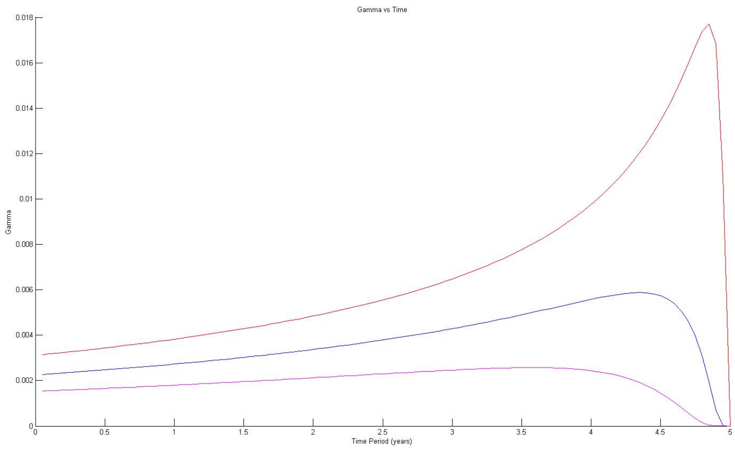
Since the delta of our convertible bond mostly depends on the call option portion of our pricing formula, it is clear that gamma also mostly depends on the call option portion of our convertible bond pricing formula.

Our gamma surface is as follows;




The shape of the gamma surface of our convertible bond pricing model resembles that of a typical call option gamma surface. The gamma is positive and fairly low for all values of the stock price up until a year from expiration. During the final year up until expiration, the gamma of the convertible bond is very high for stock prices close to the conversion price.

The following graph further illustrates this property;



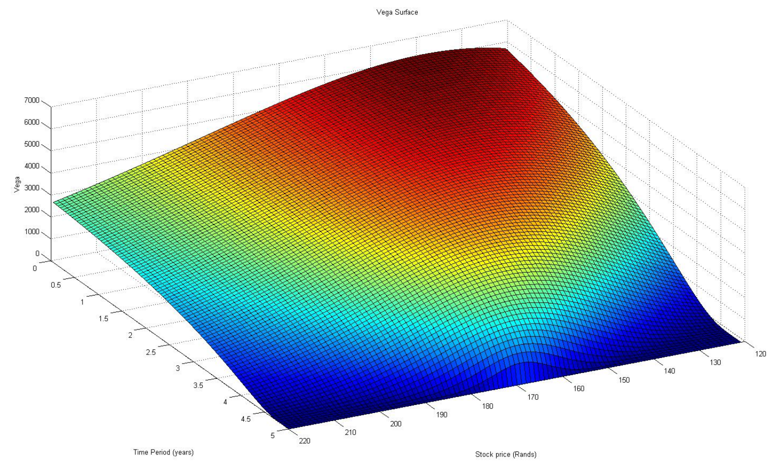

(In the Money = magenta, At the Money = blue, Out the Money = red)

## 3.6 Vega Surface

The Black Scholes pricing framework assume that the volatility of the underlying asset is constant. In practice, volatilities change over time. The vega of a portfolio is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset. If a portfolio has highly positive or highly negative vega value then the portfolio's value is very sensitive to small changes in volatility. If the vega of a portfolio is close to zero then volatility changes have relatively little impact on the value of the portfolio.

The vega of a portfolio can be changed in a similar way that the gamma of a portfolio can be changed in order to hold a neutral position. Unfortunately, in general a portfolio that is gamma neutral will not be vega neutral, and vice versa. So for a porfolio to be both gamma and vega neutral we require at least two tradded derivatives dependent on the underlying asset.

The vega surface for our convertible bond is as follows;




We note that vega is very high at the issue date when the stock price is near the conversion price. The value of the convertible bond is very sensitive to stock volatility near the conversion price because a small fluctuation of a stock price, due to volatility, near the conversion price barrier can render the option portion of the convertible bond null.

## 3.7 Rho Surface

The rho of a portfolio is the rate of change of the value of the portfolio with respect to the interest rate with all other variables remaining constant. Rho is the measure of sensitivity of the value of a portfolio subject to a change in interest rate. Our convertible bond pricing formula is a linear combination of bond price and option price therefore, to calculate rho we just need to differentiate the bond portion of the convertible bond pricing formula and add it to the rho equation of the corresponding call option. This is done as follows;

First we adjust the bond pricing formula slightly using a common geometric relation.

$IV = (C)((1 - (1 + i + cr)^{-2(T-t)})/(i + cr)) + Par(1 + i + cr)^{-2(T-t)}$

C= Nominal semi-annual coupon value

i= Effective semi-annual interest rate

Par= The principal value of the bond

T= The expiration time node

t= current time node

cr=credit risk

From this point we differentiate with respect to (T-t) and arrive at the following expression;

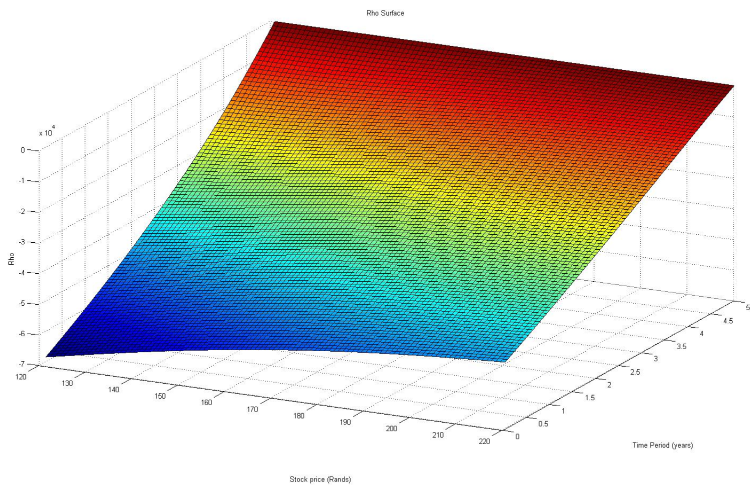
$\frac{d(IV)}{d(i)} = (C)(\frac{(2(i+cr))(T-t)(1+i+cr)^{-2(T-t)-1} - (1-(1+i+cr)^{-2(T-t)})}{(i+cr)^2}) - ...$

$...(P)(2(T-t)(1+i+cr)^{-2(T-t)-1}$

our final expression is as follows;

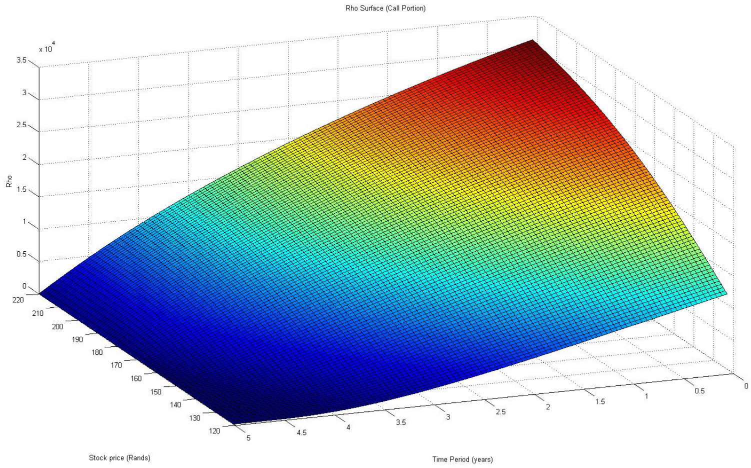
$Theta = \frac{d(IV)}{d(i)} + rho(call)$

Using this expression for the rho of our convertible bond we are able to construct the following rho surface;



Now by making a comparison of the rho surface of the option portion of the convertible bond pricing formula to that of the overall convertible bond rho we will be able to analyse the affect that the bond and option portions have on the overall rho.

The rho surface of the call portion of the convertible bond pricing formula is as follows;




From this comparison we find that the rho from the bond portion of the convertible bond has a significant effect of the overall rho of the convertible bond. Thus we can conclude that the convertible bond's sensitivity to the interest rate is strongly correlated to the bond portion of the convertible bond pricing formula.

Chapter 4

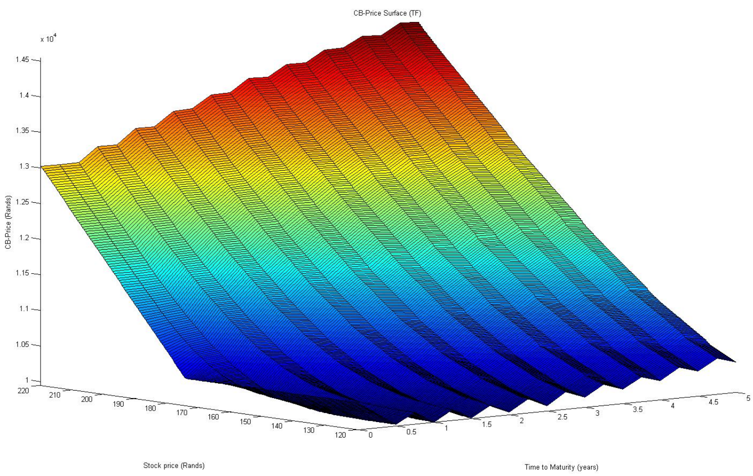
# Analysis of Tsiveriotis & Fernades Model

## 4.1 Analysis Introduction

The Tsiveriotis and Fernades model is significantly more popular than the component model that was discussed earlier. By analysing this model and comparing it to the simpler component model we hope to discover why this model is so popular.

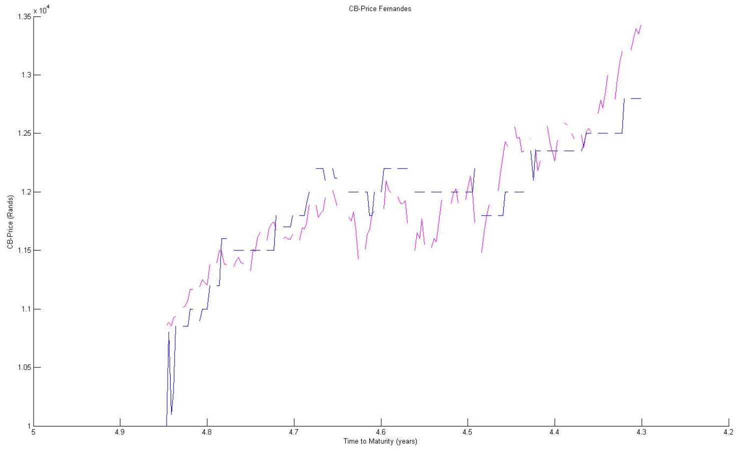
## 4.2 Price Surface

The price surface for the Tsiveriotis and Fernades model is as follows;



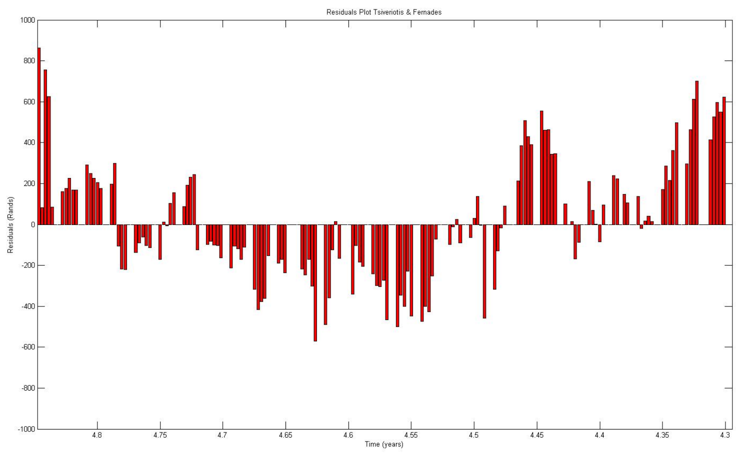

The change in value of the convertible bond due to coupons being paid is very evident in this model.

Now by fitting the Tsiveriotis and Fernades model to the empirical data we are able to observe how accurate the model is (TF-model=magenta);




The regression statistics for this model are as follows;
<u>Quadratic Regression</u>
r-squared=0.8280
adjusted r-squared=0.8255
<u>Linear Regression</u>
r-squared=0.7314
adjusted r-squared=0.7294

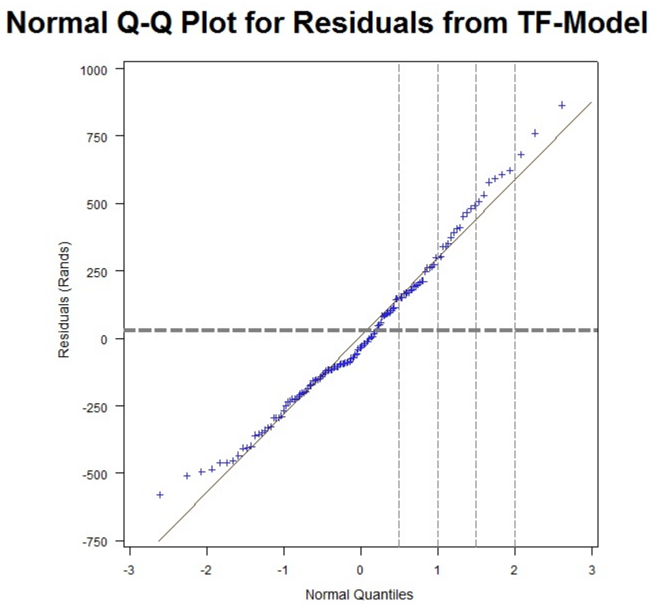
The residuals of a model are the estimates of the experimental error obtained by subtracting the observed responses from the predicted responses. The residuals of the fitted model for this time period are as follows;




Once again the model seems like an adequate fit, but we cannot make any conclusions at this stage. We need to perform further analysis on the residuals.

A well behaved model will have residuals that are roughly normally and approximately independently distributed with a mean of zero and some constant variance.

The qq plot of the residuals show strong signs of normality with a mean that is significantly close to zero;




To confirm our assumption of normality we perform a series of tests for normality. The output is as follows;

**Tests for Location: Mu0=0**

| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | 0.883827 | Pr > \|t\| | 0.3783 |
| Sign | M | -1.5 | Pr >= \|M\| | 0.8644 |
| Signed Rank | S | 183 | Pr >= \|S\| | 0.6957 |

**Tests for Normality**

| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.98064 | Pr < W | 0.0487 |
| Kolmogorov-Smirnov | D | 0.078758 | Pr > D | 0.0368 |
| Cramer-von Mises | W-Sq | 0.137545 | Pr > W-Sq | 0.0363 |
| Anderson-Darling | A-Sq | 0.769295 | Pr > A-Sq | 0.0455 |

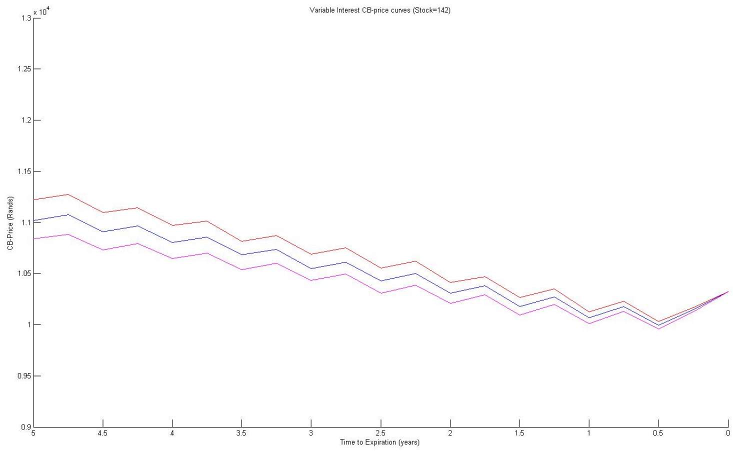
From this we are able to conclude that for a significance level of 1% we have that the residuals are normally distributed with a mean significantly close to zero. So in comparison to the simple component model, the Tsiveriotis and Fernades model describes the empirical data better at a 1%significance level .

We note that for a significance level of 5% we have that the residuals are not from a normally distributed population.

## 4.3 Interest rate sensitivity

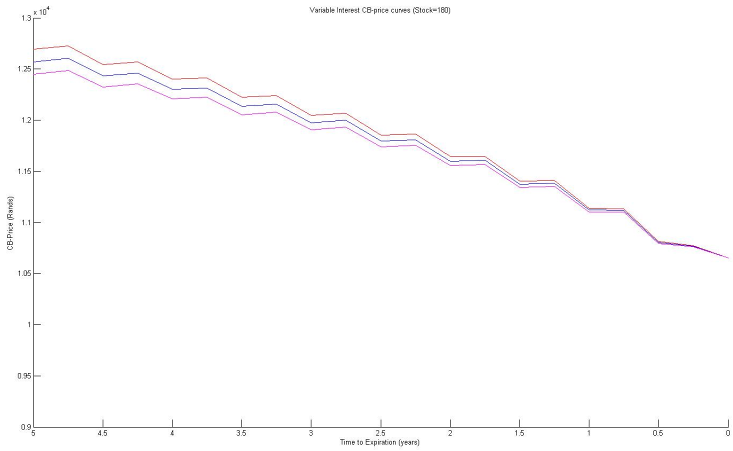
We now analyse the effect of interest rate on the value of the convertible bond over time while all other variables remain constant. For this analysis we fix the closing price of the underlying equity to R142 (Out of the money).

The convertible price curves over time at different interest rates are as follows;




(7.25% = magenta, 6.5% = blue, 5.75% = red)

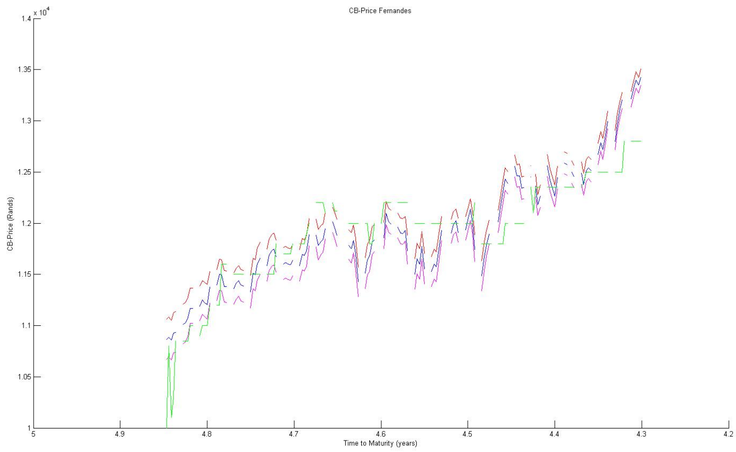
Now we will consider the effect of interest rate on the value of a convertible bond over time where the stock price is fixed at R180 (In the money).




(7.25% = magenta, 6.5% = blue, 5.75% = red)

Interests rate have a greater effect on the value of a convertible bond at the issue date opposed to the value of the convertible bond at expiration.

The plot of our model at different interest rates against the empirical data is as follows;

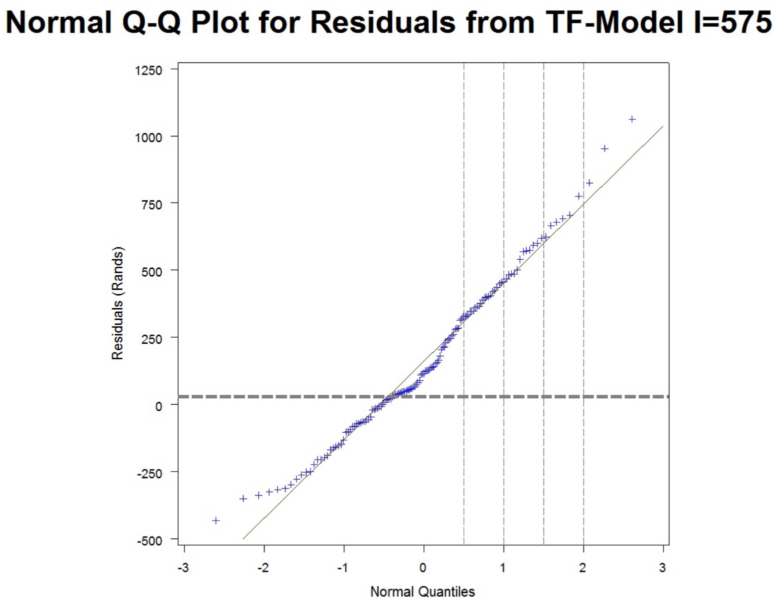



(7.25% = magenta, 6.5% = blue, 5.75% = red, Empirical=green)

From this we can see that the convertible bond value spread decreases as time progresses. We also note the inverse relationship between the value of a convertible bond and interest rate.

Now we will consider the effect of different interest rates on the fit of our model to the empirical data. By analysing the residuals we can find which interest rate choice helps the model describes the empirical data better. In order for our model to describe the empirical data as completely as possible, there must not be any structure in the residuals of our fitted model.

We will first analyse the residuals of the fitted model where the interest rate is 5.75%. The qq-plot for the residuals is as follows;




The tests and their statistics for zero mean and normality are as follows;

**Tests for Location: Mu0=0**

| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | 6.478836 | Pr > \|t\| | <.0001 |
| Sign | M | 27.5 | Pr >= \|M\| | <.0001 |
| Signed Rank | S | 2580.5 | Pr >= \|S\| | <.0001 |

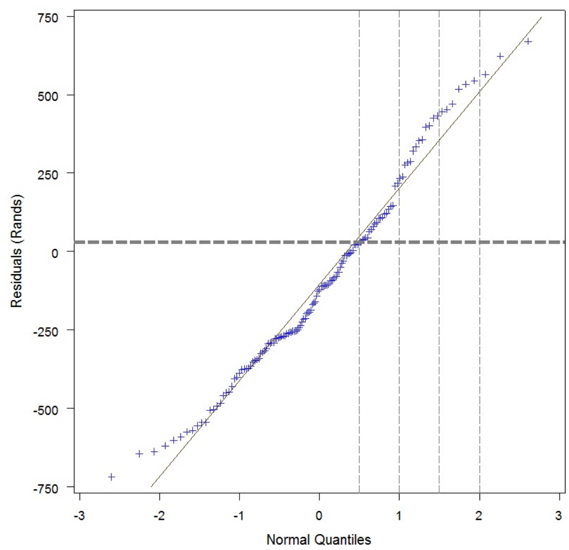
**Tests for Normality**

| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.981078 | Pr < W | 0.0542 |
| Kolmogorov-Smirnov | D | 0.087516 | Pr > D | 0.0110 |
| Cramer-von Mises | W-Sq | 0.148474 | Pr > W-Sq | 0.0245 |
| Anderson-Darling | A-Sq | 0.761688 | Pr > A-Sq | 0.0471 |

We observe that the hypothesis of zero mean is rejected at a 1% significance level. So using an interest rate of 5.75% for our Tsiveriotis and Fernandes model yields an unfavourable fit to the empirical data.

We will now analyse the residuals of the fitted model where the interest rate is 7.25%. The qq-plot for the residuals is as follows;

**Normal Q-Q Plot for Residuals from TF-Model I=725**




The tests and their statistics for zero mean and normality are as follows;

**Tests for Location: Mu0=0**

| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | 6.478836 | Pr > \|t\| | <.0001 |
| Sign | M | 27.5 | Pr >= \|M\| | <.0001 |
| Signed Rank | S | 2580.5 | Pr >= \|S\| | <.0001 |

**Tests for Normality**

| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.981078 | Pr < W | 0.0542 |
| Kolmogorov-Smirnov | D | 0.087516 | Pr > D | 0.0110 |
| Cramer-von Mises | W-Sq | 0.148474 | Pr > W-Sq | 0.0245 |
| Anderson-Darling | A-Sq | 0.761688 | Pr > A-Sq | 0.0471 |

We observe that the hypothesis of zero mean is rejected at a 1% significance level. So using an interest rate of 7.25% for our Tsiveriotis and Fernandes model yields an unfavourable fit to the empirical data.

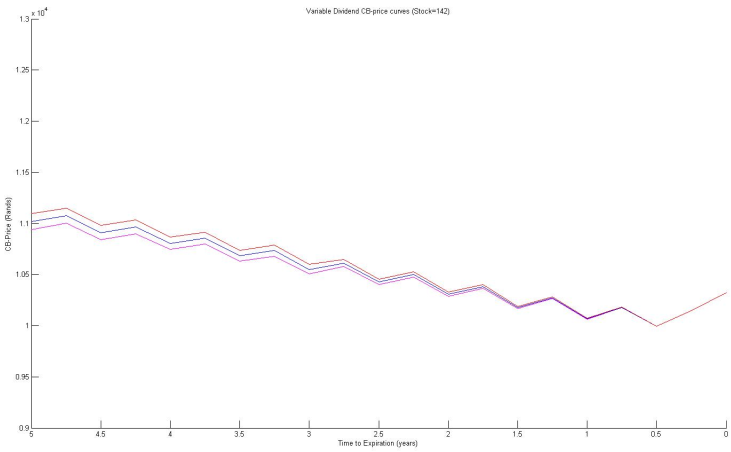
Thus is conclusion we find that our initial assumption of a yearly effective interest rate of 6.5% yields the best fit of our Tsiveriotis and Fernandes model to the empirical data, in comparison to the interest rates of 5.75% and 7.25%.

## 4.4 Dividend rate sensitivity

Dividends are payments made by a company to their shareholders. It is a portion of the company's profits, decided by the board of directors, which is paid out. Depending on the preferences and strategy of a company, the dividend rate can be fixed or adjustable. Companies are not obligated to pay out dividends. During difficult periods of market uncertainty or recession, companies might report lower dividends or stop paying out dividends all together. In 2012 Shoprite reported a dividend payout of R3.03, which is a 19.73% increase from the previous year. Shoprite is expecting dividends of R3.72 for the upcoming fiscal year of 2013, which would represent a 22.71% increase from 2012.

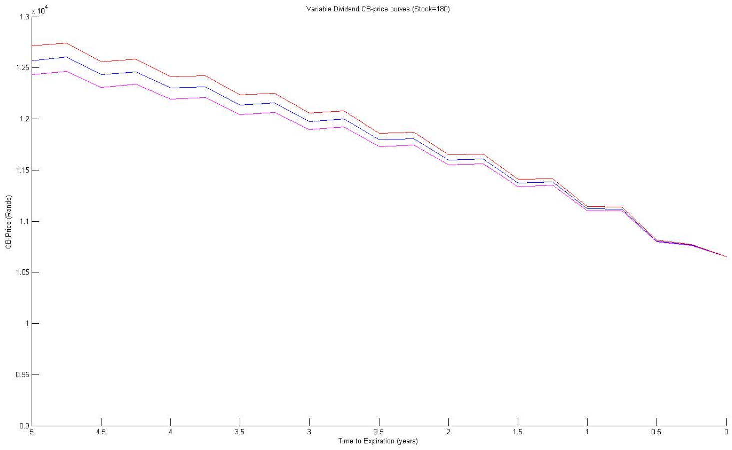
Because of the uncertainty of dividend payouts, we would like to analyse the effect that variable of dividend rate has on the value of a convertible bond under the Tsiveriotis and Fernandes model. We will first consider the case where the stock price is held constant at R142 per share. We will compare the dividend rate of 0.01496 which is 20% lower than the dividend rate 0.0187 that is used in the model. We will also make a comparison with the dividend of 0.02244, which is 20% higher than the incumbent rate.

The convertible price curves over time at different dividend rates are as follows;



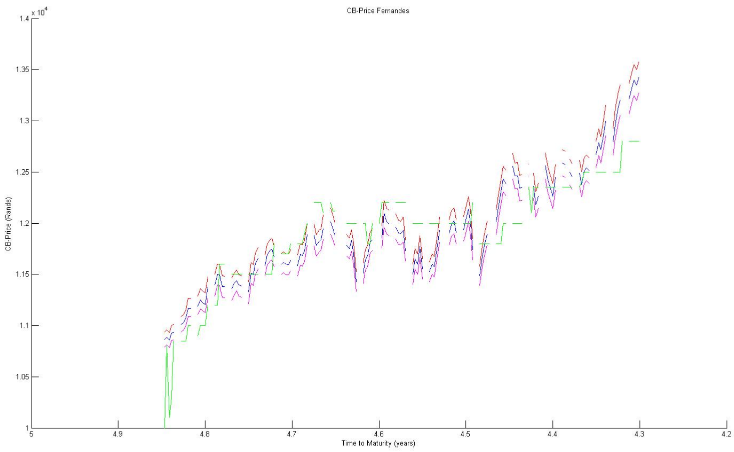

(2.244% = magenta, 1.87% = blue, 1.496% = red)

Now we will consider the effect of interest rate on the value of a convertible bond over time where the stock price is fixed at R180 (In the money).




(2.244% = magenta, 1.87% = blue, 1.496% = red)

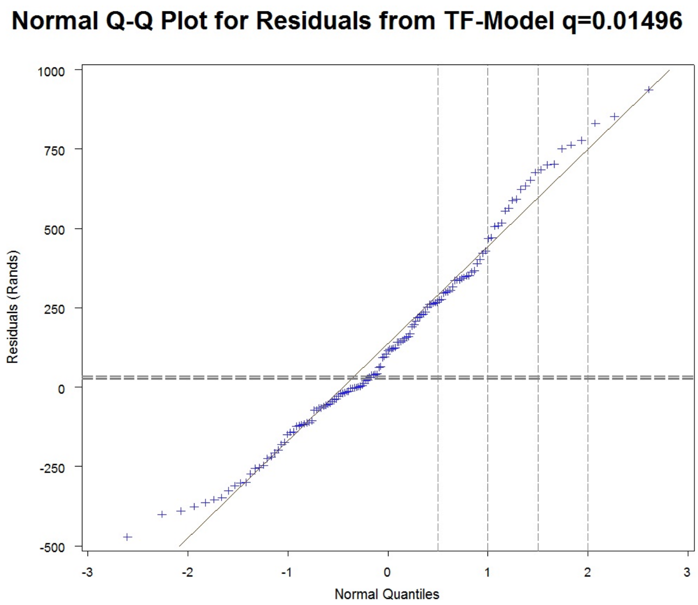
The plot of our model at different dividend rates against the empirical data is as follows;




From this we can see that the convertible bond value spread decreases as time progresses. We also note the inverse relationship between the value of a convertible bond and dividend rate.

Now we will consider the effect of different dividend rates on the fit of our model to the empirical data. By analysing the residuals we can find which interest rate choice helps the model describes the empirical data better. In order for our model to describe the empirical data as completely as possible, there must not be any structure in the residuals of our fitted model.

We will first analyse the residuals of the fitted model where the dividend rate is 1.496%. The qq-plot for the residuals is as follows;




The tests and their statistics for zero mean and normality are as follows;

Tests for Location: Mu0=0

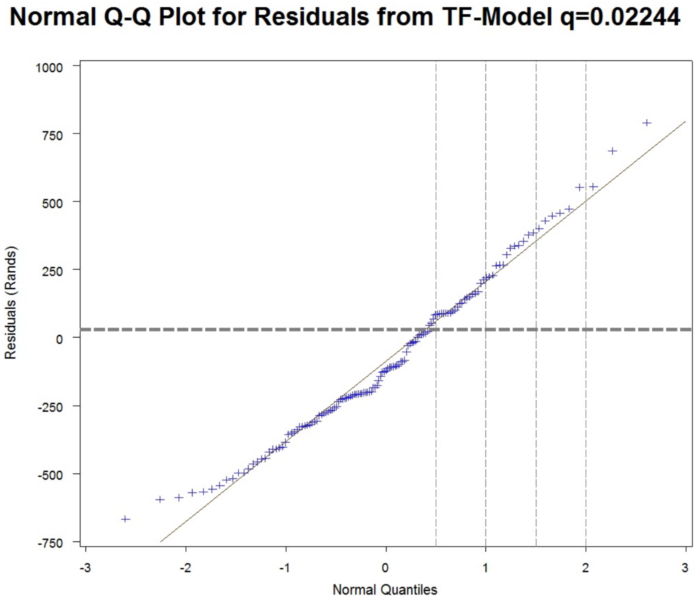
| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | 5.242889 | Pr > \|t\| | <.0001 |
| Sign | M | 16.5 | Pr >= \|M\| | 0.0061 |
| Signed Rank | S | 2044.5 | Pr >= \|S\| | <.0001 |

Tests for Normality

| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.978705 | Pr < W | 0.0306 |
| Kolmogorov-Smirnov | D | 0.082766 | Pr > D | 0.0214 |
| Cramer-von Mises | W-Sq | 0.125419 | Pr > W-Sq | 0.0503 |
| Anderson-Darling | A-Sq | 0.795502 | Pr > A-Sq | 0.0401 |

We observe that the hypothesis of zero mean is rejected at a 1% significance level. So using an dividend rate of 1.496% for our Tsiveriotis and Fernandes model yields an unfavourable fit to the empirical data.

We will now analyse the residuals of the fitted model where the dividend rate is 2.244%. The qq-plot for the residuals is as follows;




The tests and their statistics for zero mean and normality are as follows;

Tests for Location: Mu0=0

| Test | -Statistic- | | -----p Value------ | |
|---|---|---|---|---|
| Student's t | t | -3.47976 | Pr > \|t\| | 0.0007 |
| Sign | M | -16.5 | Pr >= \|M\| | 0.0061 |
| Signed Rank | S | -1679.5 | Pr >= \|S\| | 0.0002 |

Tests for Normality

| Test | --Statistic--- | | -----p Value------ | |
|---|---|---|---|---|
| Shapiro-Wilk | W | 0.980322 | Pr < W | 0.0451 |
| Kolmogorov-Smirnov | D | 0.092326 | Pr > D | <0.0100 |
| Cramer-von Mises | W-Sq | 0.136354 | Pr > W-Sq | 0.0377 |
| Anderson-Darling | A-Sq | 0.750815 | Pr > A-Sq | 0.0494 |

We observe that the hypothesis of zero mean is rejected at a 1% significance level. So using an dividend rate of 2.244% for our Tsiveriotis and Fernandes model yields an unfavourable fit to the empirical data.

Thus is conclusion we find that our initial assumption of a yearly dividend rate of 1.87% yields the best fit of our Tsiveriotis and Fernandes model to the empirical data, in comparison to the dividend rates of 1.496% and 2..244%.

# Chapter 5

# Conclusion

There are many interesting and diverse investment opportunities in the world of finance. Although many hybrid financial instruments seem complicated and even daunting at times, we find that convertible bonds are fairly simple in concept. Those who wish to invest in or issue convertible bonds should still fully understand the constraints of such a hybrid security as well as characteristics of the pricing formula used in the evaluation. The characteristics of the pricing formula can easily be identified through the use of a sensitivity report.

When fitting our models to the empirical data it is not obvious which model has the better fit. This is why it is important to perform an analysis of the model residuals to verify if the model explains the data significantly. By adjusting certain parameters and then performing a residual analysis we can observe the effect that that particular variable has on the model under consideration.

This type of analysis can be used on all of the model variables under consideration in this paper. The analysis also proves useful when calibrating the model to better fit the observed data. After model calibration, one could improve the model fit by adding more parameters.

There is no doubt about the importance of the sensitivity report when in comes to options and other derivatives. Hybrid securities that incorporate derivative like properties also need cautious analysis and much care should be taken when performing such an analysis.

# Chapter 6

# Appendix

## 6.1 Component Model Code (MatLab)

```
function [X,Y,Z]=Plotz;

 T=5; % T=term in years
 P=10000; % P=Par value
 C=0.065; % C=coupon rate convertible half yearly
 I=0.065; % I=interest rate p.a. eff
 R=59.19261276; % R=Conversion ratio (1bond:R)
 S0=148.8763; % S0=stock price at time 0
 v=0.1609; % v=volatility
 q=0.0187; % q=stock dividend rate p.a.
 cr=0.0054; % Credit risk
 ConP=P/R; % Conversion Price (K)

 %Plot inputs
 n=100; %Step size for graph
 S1=120; %Stock lower bound
 S2=220; %Stock upper bound
 tt=5; % Time range

 % Bond Pre-Calculations
 effI=(1+I)^(0.5)-1;
 effC=C/2;
 Cpn=P*effC;
 vol=v;
 q=q;
 Ic=log(1+I); % Continuous interest used in Call Price and Greeks

 % Adjusted for Credit Risk !!
 for i=1:n
 for j=1:n
 t=((tt)/n)*i;
 S0=S1+((S2-S1)/n)*j;
```

```
%The Bond Portion
An=(1-(1+effI+cr)^(-2*(T-t)))/(effI+cr);
As=(1+effI+cr)^(-2*(T-t));
InvestValue=Cpn*An+P*As; % Value of Bond portion

%The call portion
d1=(log((S0*R)/InvestValue)+(Ic-q+(vol^2)/2)*(T-t))/(vol*sqrt(T-t+eps));
d2=d1-vol*sqrt(T-t);
Nd1 = cdf('Normal',d1,0,1);
Nd2 = cdf('Normal',d2,0,1);
Nd11 = pdf('Normal',d1,0,1);

Cp=Nd1*S0*R*exp(-q*(T-t))-exp(-Ic*(T-t))*Nd2*InvestValue;
Convertible_Bond_Price=InvestValue+Cp;

%Greeks and such
Z(i,j)=Convertible_Bond_Price; %CB Price
Z1(i,j)=R*exp(-q*(T-t))*Nd1; %Delta
Z2(i,j)=(Nd11*exp(-q*(T-t)))/(S0*vol*sqrt(T-t+eps)); %Gamma
Z3(i,j)=-((R*S0*Nd11*vol*exp(-q*(T-t)))/(2*sqrt(T-t+eps))) + ...
(q*R*S0*Nd1*exp(-q*(T-t))) - (Ic*InvestValue*exp(-Ic*(T-t))*Nd2); % Theta
call
Z4(i,j)= R*S0*Nd11*sqrt(T-t)*exp(-q*(T-t)); %Vega
Z5(i,j)=InvestValue*(T-t)*exp(-Ic*(T-t))*Nd2; %Rho call
Z6(i,j)=Z3(i,j)+(Cpn/(effI+cr))*(2*log(1+effI+cr))*((1+effI+cr)^(-2*(T-t)))-
...
P*(2*log(1+effI+cr))*((1+effI+cr)^((-2)*(T-t))); %Theta
Z7(i,j)=Z5(i,j)+Cpn*((2*(effI+cr)*(T-t)*((1+effI+cr)^(-2*(T-t)-1))-...
(1-(1+effI+cr)^(-2*(T-t))))/((effI+cr)^2))-...
P*(2*(T-t)*((1+effI+cr)^(-2*(T-t)-1))); %Rho
end
end

% Plotting all Figures
figure(1)
hold on;
colormap(jet); % Set colors
x = ((tt)/n):((tt)/n):tt;
y = (S1+((S2-S1)/n)):((S2-S1)/n):S2;
[X,Y] = meshgrid(y,x);
rotate3d on; % Activate interactive mouse rotation
surf(X,Y,Z,'EdgeColor','black','FaceColor','interp','FaceLighting','phong')
axis([S1 S2 0 tt -inf inf]); % Set up axes
title('CB-Price Surface');
xlabel('Stock price (Rands)'); % Label axes
ylabel('Time Period (years)');
zlabel('CB-Price (Rands)')

% REAL DATA !!!!
```

```
[TT,SS,CBB]=REALDATAPLOTZ;
figure(1)
plot3(SS,TT,CBB,'Color','m')
hold off

....
....
....

figure(14)
hold on;
title('Gamma vs Time');
xlabel('Time Period (years)'); % Label axes
ylabel('Gamma');
plot(Y(1:n-1,74),Z2(1:n-1,74)) %At the money plot
(Y(:,90),Z2(:,90),'Color','m') %In the money
plot(Y(:,60),Z2(:,60),'Color','r') %Out the money hold off;

end
```

## 6.2 Component Model data SubCode (MatLab)

```
function [TT,SS,CBB]=REALDATAPLOTZ;

    TT=[0.153225806
    0.155913978
    0.158602151
    0.161290323
    0.163978495
    ...
    ...
    ...
    0.701612903
    0.704301075];

    CBB=[ 10000
    10800
    10100.15
    10300
    10850
    ...
    ...
    ...
    nan
    nan ];

    SS=[ 135.85
    136.5
    135.65
    137.82
    138.1
    ...
    ...
    ...
    nan
    nan ];

    end
```

## 6.3 Tsiveriotis and Fernades model Code (MatLab)

```
function [X,Y,Z]=Mcbprice;
    clear all;

    % Enter in parameters
    T=5; % Time to Maturity
    P=10000; % Par Value
    C=0.065; % Coupon Rate
    Cf=0.5; % Coupon frequency
    R=59.19261276; % conversion ratio
    S=142; % Stock price
    cr=0.0054; % Credit spread
    var=0.1609; % Volatility
    q=0.0187; % dividend rate
    I=0.065; % interest rate effective (pa)
    n=30; % time steps

    % 3D Plot parameters
    S1=120;
    S2=220;
    Sstep=0.5; % Stock Step Size
    Tstep=0.25; % Time Step Size

    % Initial Calculations
    Cp=P*C*Cf; % Coupon amount

    % Create a MeshGrid for 3D Plot
    [X,Y] = meshgrid(0:Tstep:T, S1:Sstep:S2); % X-Time Y-Stock Price
    [n1x,n2x]=size(X); [n1y,n2y]=size(Y);

    % Start Loop
    for jj=1:n2x % Columns
    for ii=1:n1y % Rows
    T=X(ii,jj); % Expiration Time
    S=Y(ii,jj); % Stock Price
    dt=T/n; % delta t
    sU=exp((I-q-0.5*(var^2))*dt)*exp(var*(dt^0.5)); % Stock up
    sD=exp((I-q-0.5*(var^2))*dt)*exp(-var*(dt^0.5)); % Stock down
    timedt=dt:dt:T; % Time steps
    NCP=floor(T/Cf); % Coupons outstanding
    CptimeI=zeros(NCP,1); % Number of timesteps to coupon times

    % Coupon indexes
    for i=1:NCP
    CPtime=i*Cf;
    [minval,minidx]=min( abs(timedt-CPtime) );
    CptimeI(i)=minidx;
```

```
end

% Initialize
i=n;
U1=zeros(i+1,i+1);
V1=zeros(i+1,i+1);
StI=zeros(i+1,1);
Ss1=zeros(i+1,i+1);
Bb1=zeros(i+1,i+1);

% Check conditions (Call payoff)
for j=0:i
Ssud=S* sUˆj * sDˆ(i-j); StI(j+1)=Ssud;
Ss1(j+1,:)=R*Ssud;
for k=0:i
Bb1(j+1,k+1)=(P+Cp);
if Ss1(j+1,k+1)¿=Bb1(j+1,k+1)
V1(j+1,k+1)=0; U1(j+1,k+1)=Ss1(j+1,k+1);
else V1(j+1,k+1)=Bb1(j+1,k+1); U1(j+1,k+1)=0;
end
end
end

% Check values at all timesteps and adjust
for i=n-1:-1:0
U2=zeros(i+1,i+1);
V2=zeros(i+1,i+1);
Ss2=zeros(i+1,i+1);
Bb2=zeros(i+1,i+1);

for j=0:i %loops over S1 nodes
Ssud=(sUˆj*sDˆ(i-j))*(S);
StI(j+1)=Ssud;
Ss2(j+1,:)=R*Ssud;

for k=0:i
Bb2(j+1,k+1)=0.25*exp(-(I+cr)*dt)*(V1(j+1,k+1)+V1(j+2,k+1)+V1(j+1,k+2)+V1(j+2,k+2))
...
+0.25*exp(-I*dt)*(U1(j+1,k+1)+U1(j+2,k+1)+U1(j+1,k+2)+U1(j+2,k+2));

CPadj=0;
% Adjust for coupons
if sum(CptimeI==i)¿0
Bb2(j+1,k+1)=Bb2(j+1,k+1)+Cp;
CPadj=Cp;
end

if Ss2(j+1,k+1)¿=Bb2(j+1,k+1)
U2(j+1,k+1)=Ss2(j+1,k+1);
V2(j+1,k+1)=0;
```

```
else
U2(j+1,k+1)=0.25*exp(-I*dt)*(U1(j+1,k+1)+U1(j+2,k+1)+U1(j+1,k+2)+U1(j+2,k+2));
V2(j+1,k+1)=0.25*exp(-(I+cr)*dt)*(V1(j+1,k+1)+V1(j+2,k+1)+V1(j+1,k+2)+V1(j+2,k+2))+CPadj;
end
end
end
U1=U2;
V1=V2;
Ss1=Ss2;
Bb1=Bb2;
end

CBValue=U2+V2;
Z(ii,jj)=CBValue;
end
end

% % Plots
figure(15)
hold on;
colormap(jet); % Set colors
rotate3d on; % Activate interactive mouse rotation
surf(X,Y,Z,'EdgeColor','black','FaceColor','interp','FaceLighting','phong');
axis([0 5 S1 S2 -inf inf]); % Set up axes
title('CB-Price Surface (TF)'); ylabel('Stock price (Rands)'); % Label axes
xlabel('Time to Maturity (years)');
zlabel('CB-Price (Rands)')

% REAL DATA !!!!
[TT,SS,CBB]=REALDATA;
figure(15)
plot3(TT,SS,CBB,'Color','m')
hold off;
end
```

## 6.4 Tsiveriotis and Fernades model data Sub-Code (MatLab)

```
function [TT,SS,CBB]=REALDATA;

    TT=[ 4.846774194
    4.844086022
    4.841397849
    4.838709677
    4.836021505
    ...
    ...
    ...
    4.298387097
    4.295698925 ];

    CBB=[ 10000
    10800
    10100.15
    10300
    10850
    ...
    ...
    ...
    nan
    nan ];

    SS=[ 135.85
    136.5
    135.65
    137.82
    138.1
    ...
    ...
    ...
    nan
    nan ];

    end
```

## 6.5 Run models and comparitive fit Super Code (MatLab)

```
% Fernandes Model
    clear all;
    [X,Y,Z]=Mcbprice;
    [TT,SS,CBB]=REALDATA;
    [rCBB cCBB]=size(CBB);

    for i=1:rCBB
    ZI(i)=griddata(X,Y,Z,TT(i),SS(i));
    end
    Residuals=ZI'-CBB;

    figure(16)
    hold on;
    plot(TT,ZI,'m-') % Model Data
    plot(TT,CBB,'b-') % Real Data
    title('CB-Price Fernandes');
    ylabel('CB-Price (Rands)'); % Label axes
    xlabel('Time to Maturity (years)');

    figure(17)
    bar(TT,Residuals,'r')
    axis([-inf inf -1000 1000]);
    title('Residuals Plot Tsiveriotis & Fernades');
    ylabel('Residuals (Rands)'); % Label axes
    xlabel('Time (years)');

    %Component model
    clear all;
    [X,Y,Z]=Plotz;
    [TT,SS,CBB]=REALDATAPLOTZ;
    [rCBB cCBB]=size(CBB);

    for i=1:rCBB
    ZI(i)=griddata(Y,X,Z,TT(i),SS(i));
    end
    Residuals=ZI'-CBB;

    figure(18)
    hold on;
    plot(TT,ZI,'m-') % Model Data
    plot(TT,CBB,'b-') % Real Data
    title('CB-Price Component');
    ylabel('CB-Price (Rands)'); % Label axes
    xlabel('Time (years)');
    ...
    ...
```

## 6.6 Residual Analysis Component Model Code (SAS)

```
data Failures2;
    input Error;
    label Error = 'Residuals (Rands)';
    datalines; 759.62
    -19.327
    651.92
    ...
    ...
    ...
    534.97
    608.97
    ;
    run
    symbol v=plus;
    title 'Normal Q-Q Plot for Residuals from Component-Model';
    proc univariate data=Failures2 all;
    qqplot Error / normal(mu=est sigma=est)
    square
    href=0.5 1 1.5 2
    vref=25 27.5 30 32.5 35
    lhref=4
    lvref=4;
    run;
```

## 6.7 Residual Analysis Tsiveriotis and Fernades model Code (SAS)

```
data Failures;
    input Error @@;
    label Error = 'Residuals (Rands)';
    datalines; 862.06
    82.801
    757.03
    ...
    ....
    ...
    549.94
    623.64
    ;
    run
    symbol v=plus;
    title 'Normal Q-Q Plot for Residuals from TF-Model';
    proc univariate data=Failures all;
    qqplot Error / normal(mu=est sigma=est)
    square
    href=0.5 1 1.5 2
    vref=25 27.5 30 32.5 35
    lhref=4 lvref=4;
    run;
```

# Bibliography

[1]

[1]Convertible Bonds: An Introduction. (2010). Retrieved May 19 2012. http://www.investopedia.com/articles/ViewArticlePrintable.aspx?url=%2farticles%2f01%2f052301.asp

[2]How to Improve the BS Assumptions. (1997). Retrieved May 19 2012. www.umsl.edu/~fungh/BSMODEL.PPT

[3] Prime Interest Rate History. (2012). Retrieved May 19 2012. http://liberta.co.za/blog/prime-interest-rate-in-south-africa-current-and-historical/

[4] Fin24 Shoprite Holdings Limited. (2012). Retrieved May 20 2012. http://www.fin24.com/Company/Shoprite-Holdings-Ltd

[5] Bloomberg Shoprite holdings Ltd. (2012). Retrieved May 20 2012. http://www.bloomberg.com/quote/SHP:SJ

[6] What are the assumptions behind the Black-Scholes model? (No date). Retrieved May 20 2012. http://www.maxi-pedia.com/Black+Scholes+model+assumptions

[7] Get Acquired With Bond Price/Yield Duo. (2010). Retrieved May 20 2012. http://www.investopedia.com/articles/bonds/07/price_yield.asp#axzz1vsOo4fDC

[8] Black, F. & Myron, S. (1973) The Pricing of Options and Corporate Liabilities. Journal of Political Economy 81 (3): 637–654.

[9] Banks, E. & Siegel, P. (2006) The options applications handbook: hedging and speculating techniques for professional investors. McGraw-Hill Professional.

[10] Listing Document dated 24 May 2012: SHOPRITE INVESTMENTS LIMITED. Retrieved May 28 2012. http://www.shopriteholdings.co.za/files/1019812640/Investor_Centre_Files/Shoprite%20Listing%20Docu

[11] Convertible Monthly (August 2012). Merrill Lynch Global Convertibles Research Group

[12] Convertible Bonds: An Introduction to the Asset Class. RMF Investment Products

[13] Convertible Bonds: An Introduction. Retrieved August 2012.
http://www.fam.ch/en/knowledge/theory/cb_-_an_introduction.html

[14] SA-PD 5y IRS spread: the trade still has potential. Retrieved August 2012. http://www.research.hsbc.com/midas/Res/RDV?p=pdf&key=lnEiFiDLNJ&n=309168.PDF

[15] The Mechanics of convertible bonds. Retrieved June 2012. http://www.google.com/url?sa=t&rct=j&c

[16] TeachingNote_ConvertibleBonds. Retrieved June 2012.
http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0CCIQFjAA&url=http

[17] Zadikov, A. ZDKARI001. (November 2010). Methods of Pricing Convertible Bonds.

[18] Shoprite Listing Document May 2012_FINAL. Retrieved June 2012. Retrieved June 2012.

[19] Hull, J.C. 2012. Options, Futures And Other Derivatives (8th edition). Essex: Pearson Education.

[20] Sharenet: Java Technical Analysis Charts. Retrieved December 2012.
http://www4.sharenet.co.za/charts/index.php?load=SHPCB

[21] Are the model residuals well behaved? Retrieved December 2012.
http://www.itl.nist.gov/div898/handbook/pri/section2/pri24.htm

[22] Tsiveriotis & Fernandes. (1998).Valuing Convertible Bonds with Credit Risk.

[23] Maseko, S. (2010). Analysis and Implementation of Convertible Bond Models.

[24] Landskroner, Y & Raviv, A. (2002). The Valuation of Inflation-Indexed and FX Convertible Bonds.

[25] Factors affacting dividend policy. Retrieved Devember 2012.
http://wiki.answers.com/Q/Factors_affecting_dividend_policy

[26] Understanding the Dividend Yield on a stock. Retrieved Devember 2012.
http://moneyover55.about.com/od/howtoinvest/a/dividendyields.htm

[27] Are the model residuals well-behaved?. Retrieved Devember 2012.
http://www.itl.nist.gov/div898/handbook/pri/section2/pri24.htm

[28] ft.com/marketdata. Retrieved Devember 2012.
http://markets.ft.com/research/Markets/Tearsheets/Forecasts?s=SHP:JNB